Practiccal 14

Generate 100 observations from 2 normal distribution with density

$$f_m(x) = 0.4N(\mu = 10, \sigma = 2) + 0.6N(\mu = 15, \sigma = 2.5)$$

Oblain MLE for mu_1 and mu_2 and p_1 , p_2

Workout

generating random variate

```
# mixture components
mu.true    = c(10, 15)
sigma.true = c(2, 2.5)

# determine Z_i
Z = rbinom(500, 1, 0.4)
# sample from mixture model

X <- rnorm(100, mean=mu.true[Z+1], sd=sigma.true[Z+1])
print(X)
```

```
##   [1] 10.201093 13.597627 11.432476 10.153866 11.311358 15.969057 10.668317
##   [8] 10.828780  7.532927 14.510449 10.536829 19.651289 10.940751 10.130748
##  [15] 12.732905 11.440565 12.901526 16.210629 11.075873  9.013398 12.791875
##  [22]  7.290939  9.865919  7.445358 11.573432  9.107094  7.329995 19.129570
##  [29] 10.874440 14.374692 10.147761 10.285335 15.334998 10.548552 18.421631
##  [36] 10.281123 17.855796 16.922384 11.234347 10.148474 10.181201 13.169021
##  [43] 10.508698  9.680684 14.853992 12.343916 16.025497  9.531387  8.406352
##  [50] 13.401897 14.128570  7.612134  8.504270 15.982776  7.445584 16.970596
##  [57]  9.638769 12.349774 14.158122  8.857359 16.979831 10.687504  7.308056
##  [64] 12.821057 11.094987  8.866011 12.305684 16.990914 15.649164 18.110833
##  [71]  9.304076 14.624271 13.624914 12.676885 12.714292  6.950973  8.133042
##  [78] 10.566859 15.040114 12.019144 20.227226  9.880884 15.071598  9.006461
##  [85]  7.574296 14.819312 18.254019 12.798585 10.853834 12.843832 12.955207
##  [92]  8.901808 17.235890  5.699043 11.867732 15.252310  8.720644  7.509049
##  [99]  7.961295  8.497768
```

Now estimate function

```
compute.log.lik <- function(L, w) {  #to compute log likelihood
  L[,1] = L[,1]*w[1]
  L[,2] = L[,2]*w[2]
  return(sum(log(rowSums(L))))
}


mixture.EM <- function(w.init, L) {
```

```r
  w.curr <- w.init

  # store log-likehoods for each iteration
  log_liks <- c()
  ll       <- compute.log.lik(L, w.curr)
  log_liks <- c(log_liks, ll)
  delta.ll <- 1

  while(delta.ll > 1e-5) {
    w.curr   <- EM.iter(w.curr, L)
    ll       <- compute.log.lik(L, w.curr)
    log_liks <- c(log_liks, ll)
    delta.ll <- log_liks[length(log_liks)]  - log_liks[length(log_liks)-1]
  }
  return(list(w.curr, log_liks))
}

EM.iter <- function(w.curr, L, ...) {

  # E-step: compute E_{Z|X,w0}[I(Z_i = k)]
  z_ik <- L
  for(i in seq_len(ncol(L))) {
    z_ik[,i] <- w.curr[i]*z_ik[,i]
  }
  z_ik     <- z_ik / rowSums(z_ik)

  # M-step
  w.next   <- colSums(z_ik)/sum(z_ik)
  return(w.next)
}
```

Since the mixture components are fully specified, for each sample X_i we can compute the likelihood $P(X_i|Z_i = 0)$ and $P(X_i|Z_i = 1)$. We store these values in the columns of L

```r
L = matrix(NA, nrow=length(X), ncol= 2)
L[, 1] = dnorm(X, mean=mu.true[1], sd = sigma.true[1])
L[, 2] = dnorm(X, mean=mu.true[2], sd = sigma.true[2])
```

Performing EM

```r
ee <- mixture.EM(w.init=c(0.5,0.5), L)
print(paste("Estimate = (", round(ee[[1]][1],2), ",", round(ee[[1]][2],2), ")", sep=""))
```

```
## [1] "Estimate = (0.61,0.39)"
```